1. **Dimensional analysis (5 pts):**
Suppose a tropical convective system delivers rain at a rate of 5 mm/hr for 4 hours. How much total condensation of water mass per unit area does this rain event produce?

2. **First law of thermodynamics in log pressure coordinates (5 pts):**
Show that the dry thermodynamic tendency equation in isobaric coordinates
\[
\frac{\partial}{\partial t} + v_h \cdot \nabla_h T - S_p \omega = \frac{J}{c_p}; \quad S_p = -T \frac{\partial \ln \theta}{\partial p}
\]
[see Holton 3.6]
can expressed in log pressure coordinates, i.e., \( z^* = -H \ln(p/p_s) \), as:
\[
\frac{\partial}{\partial t} + v_h \cdot \nabla_h \left( \frac{\partial \Phi}{\partial z^*} \right) + w^* N^2 = \frac{\kappa J}{H}
\]
[see Holton 8.45]
Here, \( N \) is the Brunt-Väisälä frequency, which is found to vary slowly with height in the troposphere: the log pressure formulation can thus be simplified by approximating \( N \) as constant.

3. **Richardson number (5 pts):**
In the lecture on scale analysis, we noted the importance of the Rossby number, defined as the ratio of advective (or inertial) force to the Coriolis force, i.e., \( \text{Ro} = \frac{U}{fL} \). Another important dimensionless number is the Richardson number, which is defined as the ratio of the buoyancy and inertial forces, i.e., \( \text{Ri} = \left( \frac{HN}{U} \right)^2 \).

a) Estimate \( \text{Ri} \) for nonprecipitating synoptic scale motions in the Tropics for the characteristic scales assumed in the scale analysis.
b) How would you expect \( \text{Ri} \) to change for precipitating synoptic scale systems in the Tropics? [Hint: Think about the definition of Brunt-Väisälä frequency and its relationship to stability.]
4. **Held-Hou model of the Hadley circulation (5 pts):**

Consider the Held-Hou model Hadley cell width,

\[ Y = \left( \frac{5\Delta \theta_0 H}{3\Omega^2 \theta_0} \right)^{1/2}. \]

For a planet, X, with a length of day 20% longer than Earth, a mass 3x as large as Earth but with the same planetary density, and an incoming stellar energy flux at the top of the atmosphere 1/2 as large, find \( Y_X \), expressed in terms of Earth quantities. Assume the tropospheric depth and equator-to-pole temperature gradient of planet X are the same as on Earth, and treat \( \theta_0 \) as X’s planetary blackbody temperature.