Equatorially-trapped waves
[Inertia-Gravity, Rossby, Mixed Rossby-Gravity, Kelvin waves]:
*A Brief Summary**

We begin with the Shallow Water Equations (SWE), which you'll recall are simplified forms of the governing equations subject to the approximations discussed in lecture. We can consider either a two-layer version, as in class, or a continuously-stratified version. The former is sometimes referred to as "reduced gravity", since gravity g is replaced by $g' = g\frac{\rho_l - \rho_u}{\rho_l}; \rho_l > \rho_u$, i.e., g scaled by the density difference in the lower and upper layers divided by the density in the lower layer. For the latter, we need to solve a vertical structure equation: from this equation, we obtain an equivalent depth, H_n: you can think of H_n, which in general depends on the vertical wavelength of the wave type considered, replacing the actual "depth" of the lower fluid layer in the two-layer version of the SWE, while g is used instead of g'.

The SWE comprise 3 equations: the two horizontal components of momentum and continuity. For obtaining wave solutions, we consider perturbations with respect to some background state: in lecture, we assumed the background state to be a fluid at rest with a specified mean depth. For the Tropics, it is also common to assume a constant easterly zonal flow.

Let's consider first the case for which meridional velocity is assumed to be explicitly nonzero. We assume separable form solutions of the form $e^{i(\omega t + kx)}\hat{F}(y)$. Here, ω is frequency and k is zonal wavenumber. The y-dependence is "special" because of the dependence of the Coriolis parameter on latitude. Note that each of the 3 equations has it's own $\hat{F}(y)$. After some manipulation, we derive a 2nd order differential equation for $\hat{v}(y)$. This equation represents an eigenvalue equation for the meridional structure: its general solution consists of a Gaussian—decaying exponential in the limit of large displacement from the equator, which is necessary for being "equatorially-trapped"—multiplied by a set of polynomials called Hermite polynomials. Recall also that this equation has a cubic ω dependence: thus, in general, each eigenmode is "threefold degenerate", i.e., for every meridional mode number *l*, there are three solutions.

The "high-frequency" solutions comprise two inertia-gravity wave modes, [*]one mode with eastward phase propagation, the other with westward phase propagation. The "low-frequency" solutions comprise westward phase-propagating Rossby waves.

Special consideration is required for $l=0$: a step in the derivation requires one ω root to be excluded, so the resulting eigenvalue equation for $l=0$ is only quadratic in ω. One of the roots is eastward and resembles eastward-propagating inertia-gravity waves. The other root is the Mixed Rossby-Gravity wave: at low zonal wavenumber k (small wavelength), it resembles westward-propagating inertia-gravity waves, while at high zonal wavenumber (large wavelength), it resembles Rossby waves.

It is also possible to consider a case for which the meridional velocity is explicitly zero. Under this assumption, the meridional solution is simply a Gaussian (again, the requirement of trapping excludes the possibility of an exponentially-increasing solution). This solution comprises the Kelvin wave. (Note that the Kelvin wave corresponds mathematically to a solution with $l = -1$.)

These waves can be summarized on a zonal wavenumber-frequency plot (k and ω axes), as shown in lecture. A Wheeler-Kiladis diagram is related to such a graphical depiction in that is shows how the spectral power of observed tropical variability, a measure of the amount of variability at different combinations of k and ω, compares to the idealized "dry" SWE solutions described above.

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*Appendices A, B, and C of Chapter 5 in the Comet *Introduction to Tropical Meteorology* on-line textbook provide a thorough overview of the derivation of the SWE, the solutions for equatorially-trapped waves, and Wheeler-Kiladis diagrams.