Equation of State For Moist Air

Dry air is an idealization, while water is of primary importance for the thermodynamics of the atmosphere, i.e., water is responsible for much of the interesting behavior in Earth’s atmosphere.

The IGL for $\text{H}_2\text{O}$ vapor is:

$$p_v \alpha_v = R_v T = e \alpha_v$$

$e$ is the commonly used symbol for partial pressure of water vapor; it is also called vapor pressure.

$$R_v = \frac{R^*}{m_v} \implies m_v \approx 18 \text{ kg kmol}^{-1} ; R_v \approx 462 \text{ J (kg K)}^{-1}$$
Mixture of dry air and vapor

From Dalton’s Law of Partial Pressures, the total pressure $p$ for a mixture of dry air and vapor is:

$$p = p_d + e$$

Thus, the IGL for the mixture of dry air and vapor (or “moist air”) is:

$$p = (\rho_d R_d + \rho_v R_v)T$$

We can express the density of the mixture as:

$$\rho = \rho_d + \rho_v \equiv \frac{\text{mass of dry air + mass of vapor}}{\text{volume}}$$
Mixing ratio and specific humidity

We will often use the ratio of vapor density to dry air density, which is called the **mixing ratio**, \( w \):

\[
w = \frac{\rho_v}{\rho_d}
\]

Note that mixing ratio is a conserved quantity as long as no phase changes occur.

Typically, \( w \) is expressed in units of “g of vapor per kg of dry air.” The range of \( w \) values within the atmosphere is \(~\)a few g kg\(^{-1}\) in cool, dry air masses to \(~20\) g kg\(^{-1}\) in moist, tropical air masses.

A related quantity is **specific humidity**, \( q \):

\[
q = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_d + \rho_v}
\]
Virtual temperature (I)

Consider one form of the IGL for moist air:

\[ p = (\rho_d R_d + \rho_v R_v)T \]

We can rewrite this (after some algebra!) as:

\[ p = \rho R_d \left[ 1 + q \left( \varepsilon^{-1} - 1 \right) \right] T \]

On the other hand, we can also write the IGL as:

\[ p = \rho R_m T \quad \text{[} R_m \text{ is the gas constant for moist air.]} \]

Comparing the two forms of the IGL for moist air gives:

\[ R_m = R_d \left[ 1 + q \left( \varepsilon^{-1} - 1 \right) \right] \]
Virtual temperature (II)

A more common approach is to define

\[ p = \rho R_d T_v \]

where \( T_v \) is the **virtual temperature**:

\[ T_v = \left[ 1 + q \left( \frac{e^{-1}}{-1} \right) \right] T \]

The virtual temperature can be thought of as the temperature a dry parcel would have to have such that the density of this parcel would be equivalent to the density of the moist parcel, assuming the same pressure. Virtual temperature is always \( \geq \) the actual temperature.

Consider a volume of dry air (molar mass \( \sim 29 \) kg/kmol) at temperature \( T \)...

Replacing a fraction of the dry air molecules by vapor (molar mass \( \sim 18 \) kg/kmol) at the same temperature reduces the density....

So virtual temperature \( T_v \geq T \) represents temperature which the original dry air sample would need to have for its density to equal that of the moist air sample.